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FSK SPECTRA GENERATION

FINAL REPORT

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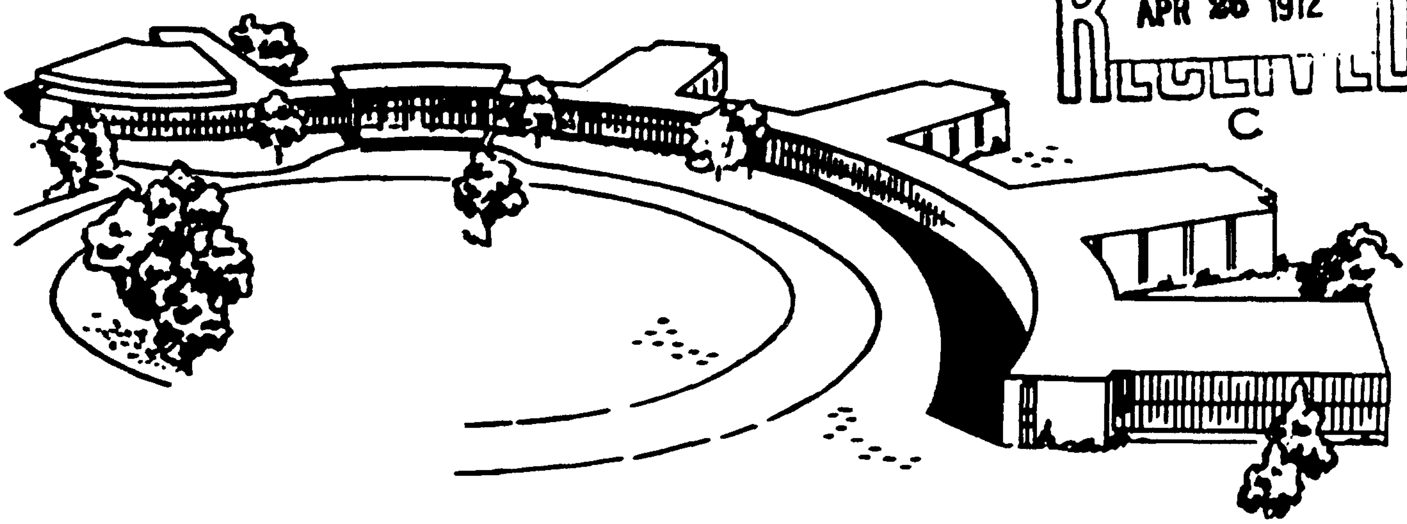

THE UNIVERSITY of TENNESSEE SPACE INSTITUTE

Tullahoma, Tennessee

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by

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13. ABSTRACT

The results of a study of a phase locked modulation technique applicable to phase coherent FSK are presented. This technique employs a continuous phase, frequency shifted, phase locked loop to produce controlled, non-instantaneous, frequency transitions between the mark and space frequencies. Operational and spectral analyses have been conducted, and spectral data have been calculated for the modulated carrier signal and its corresponding effective modulating signal. It is shown that effective modulation wave-shaping can be achieved and that the corresponding signal bandwidths can be controlled within certain limits by variation of the loop parameters, undamped natural frequency and open loop gain. Representative data are presented, corresponding to a selected class of loop parameters, as a function of modulation index and the ratio of frequency transition time to bit length.

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Phase Coherent FSK						
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Frequency-Shift Modulation						
FSK Spectrum						

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FSK SPECTRA GENERATION

Introduction

A preliminary study of modulation techniques applicable to phase coherent FSK has been conducted under Contract Number N00014-72-C-0171 with the Office of Naval Research. Results of operational and spectral analyses conducted during this study are reported here along with presentation of representative spectral data. Attention is restricted to binary FSK. This effort has dealt primarily with the determination of the spectral content of frequency shift modulated carrier signals generated by means of a continuous phase, frequency shifted, phase locked loop (PLL) which results in a controlled frequency transition (CFT) between mark and space frequencies. Additionally, attention has been given to determination of the frequency spectrum corresponding to the effective modulating signal; that is, the demodulated carrier signal.

It is well known that a reduction in signal bandwidth can be achieved by low pass filtering or otherwise shaping the rectangular modulating or keying waveform which is characteristic of FSK [1,2,3]*. In non-coherent FSK the modulating signal can be passed through a low pass filter, the

*Numbers in brackets refer to similarly numbered references in the Bibliography.

filtered signal then being employed to frequency modulate a voltage controlled oscillator (VCO). However in systems employing coherent FSK, such as some VLF communications systems [4], the aforementioned method is not applicable. A method applicable to coherent FSK employs phase lock techniques [5] to provide effective low pass filtering of the modulating signal [6]. The shape of the effective modulating signal can be controlled, within limits, by means of the phase locked loop parameters; thus, the reason for consideration of PLL modulation wave-shaping and the spectra generated by such a technique.

Knowledge of the spectral characteristics of CFT-FSK signals is not only useful in signal design and subsequent design of transmitting and receiving systems, but also in the specification and design of instrumentation employed to test and evaluate such systems. Of particular interest here is the effect of the frequency transition time upon the spectral distribution of energy and the bandpass characteristics required for transmission of the spectrum. PLL parameter values are selected which yield a smooth transition with small overshoot and settling time. A ratio of frequency transition time to bit length is defined in terms of the PLL parameters pertinent to this application. CFT-FSK analyses and results of spectral calculations are presented here and compared for various values of modulation index and a selected class of wave-shaping parameters.

PLL Response to a Frequency Step

The PLL configuration under consideration here is shown in Figure 1, along with pertinent defining equations. With reference to this figure, it is assumed for simplicity that the PLL reference signal (input) is derived from two standard frequency sources, the mark and space frequencies for binary FSK. Selection of a mark or space frequency is prescribed by a rectangular keying wave. PLL operational assumptions are: (1) the frequency shift of the PLL input signal is a continuous phase instantaneous shift, initiated at time instants of zero phase coincidence between the mark and space frequencies, and (2) the PLL operates within its hold-in range at all times. The rate of change of frequency at the output of the VCO, due to a shift in frequency at the PLL input, can be controlled by means of the PLL's open loop gain and an appropriate selection of the break frequencies associated with the loop's phase lag filter.

For purposes of determining the PLL response to an instantaneous shift in frequency of the input signal, a frequency shift from ω_1 to ω_2 is considered such that

$$\omega_1 + \Delta\omega = \omega_c = \omega_2 - \Delta\omega, \tag{1}$$

where ω_c is a ficticious carrier frequency centered between the mark and space frequencies, ω_1 and ω_2. Also, $\Delta\omega$ is the steady state frequency deviation from the carrier frequency. A unit amplitude PLL input signal for such a frequency shift can be represented by

$$v_i(t) = \sin[\omega_c t + \varphi_i(t)] \qquad (2)$$

where

$$\varphi_i(t) = \begin{cases} -\Delta\omega t, & t < o \\ +\Delta\omega t, & t > o. \end{cases}$$

That is, the PLL is assumed locked to frequency ω_1 and is excited at time equal zero by applying a frequency step or phase ramp. The VCO output signal can be expressed as

$$v(t) = \sin[\omega_c t + \varphi(t)], \; t \geq o. \qquad (3)$$

It is assumed that the VCO natural frequency (open loop frequency) is equal to the carrier frequency in order to achieve quarter-wave symmetry of the modulating waveform.

The instantaneous phase deviation from the linearly increasing phase, $\omega_c t$, is found by solution of the PLL

system equation

$$\dot{\varphi}(t) - K \int_{o}^{t} [\dot{\varphi}_i(x) - \varphi(x)]h(t-x)\, dx = \dot{\varphi}(o). \qquad (4)$$

Here, h(t) is the impulse response of the loop's phase lag filter, as shown in Figure 1. From the steady state solution of the loop's system equation, the initial conditions can be found as

$$\varphi(o) = \Lambda\omega/K \qquad (5)$$

and

$$\dot{\varphi}(o) = -\Lambda\omega \qquad (6)$$

Defining

$$K = k_o k_c \qquad (7)$$

$$\omega_n = \sqrt{K/T_a} \qquad (8)$$

$$\delta = \frac{KT_b + 1}{2T_a\omega_n} \qquad (9)$$

$$\alpha = \delta\omega_n \tag{10}$$

$$\beta = \omega_n \sqrt{1 - \delta^2} \tag{11}$$

$$\varphi_s = \Lambda\omega/K \tag{12}$$

and

$$M = 2\sqrt{\frac{1 - 2\delta\omega_n/K - (\omega_n/K)^2}{1 - \delta^2}}, \tag{13}$$

the solution of (4) for the underdamped case yields

$$\varphi(t) = \Lambda\omega t + \frac{\Lambda\omega}{\omega_n} M \exp(-\alpha t) \sin(\beta t + \theta) - \varphi_s \tag{14}$$

where

$$\theta = \arctan\left[\frac{\omega_n/K\sqrt{1 - \delta^2}}{\delta\omega_n/K - 1}\right].$$

Here, δ, ω_n and K are respectively the damping factor, the loop natural undamped frequency and the open loop gain.

Taking the first time derivative of (14) yields the instantaneous frequency deviation from the carrier frequency,

$$\dot{\phi}(t) = \Lambda\omega\{1 + M \exp(-\alpha t) \sin(\beta t + \gamma\}, \qquad (15)$$

where

$$\gamma = \arctan\left[\frac{-\sqrt{1-\delta^2}}{\delta - \omega_n/K}\right].$$

Equations 14 and 15 were derived for a up-shift in frequency, from ω_1 to ω_2. For a down-shift from ω_2 to ω_1, these expressions can be made applicable by changing $+\Delta\omega$ to $-\Lambda\omega$.

A plot of the instantaneous frequency of (15) represents an effective modulation or keying waveform, as illustrated in Figure 2. It can be seen that the shape of the waveform is determined by the parameters δ and ω_n/K. Selection of the value of these parameters is primarily based upon the desire to minimize signal bandwidth for a given frequency transition time. This suggests a smooth transition accompanied by small overshoot and settling time, such as the transitions produced by $0.7 \leq \delta \leq 1$ and $\omega_n/K = 2\delta$ (due to physical realizability considerations, $0 < \omega_n/K \leq 2\delta$).

Spectral Analysis

In the following, it is assumed that a periodic modulating signal of frequency $\omega = 2\pi/T$ is employed.

The values of ω_1/ω and ω_2/ω are even integer numbers, and the modulation index, $m = \Delta\omega/\omega$, is restricted to integer values, as required by the first PLL operational assumption. These conditions are sufficient to achieve phase coherent mark and space transmissions, and continuous phase frequency shifts of the PLL input signal.

In terms of the dimensionless quantities, $\tau = \omega t$ and $\Omega = \omega_c/\omega$, the VCO output over one period of the keying wave can be expressed as

$$v(\tau) = \sin(\Omega\tau)\cos[\Phi(\tau)] + \cos(\Omega\tau)\sin[\Phi(\tau)], \tag{16}$$

where

$$\Phi(\tau) = \begin{cases} \varphi(\tau) & , 0 \le \tau \le \pi \\ -\varphi(\tau) - m\pi, & \pi \le \tau \le 2\pi \end{cases} \tag{17}$$

and

$$\varphi(\tau) = m\{\tau + \frac{\omega}{\omega_n} M \exp(-\frac{\alpha}{\omega}\tau)\sin(\frac{\beta}{\omega}\tau + \theta) - \frac{\omega}{\omega_n}\frac{\omega_n}{K}\}. \tag{18}$$

The Fourier series expansion for the VCO output or modulated carrier can be expressed as

$$v(\tau) = C_o \sin(\Omega\tau) + D_o \cos(\Omega\tau)$$

$$+ \sum_{n=1}^{\infty} |C_n| \{\sin[(\Omega+n)\tau + \theta_n] + \sin[(\Omega-n)\tau - \theta_n]\}$$

$$+ \sum_{n=1}^{\infty} |D_n| \{\cos[(\Omega+n)\tau + \varphi_n] + \cos[(\Omega-n)\tau - \varphi_n]\}. \tag{19}$$

The complex Fourier coefficients are given by

$$C_n = \frac{1}{\pi} \int_0^{\pi} \cos[\varphi(\tau)] \exp(-jn\tau)\, d\tau \tag{20}$$

where

m = even number, n = even number

m = odd number, n = odd number,

and

$$D_n = \frac{1}{\pi} \int_0^{\pi} \sin[\varphi(\tau)] \exp(-jn\tau)\, d\tau \tag{21}$$

where

m = even number, n = odd number

m = odd number, n = even number.

The complex spectrum is therefore a function of m, δ, ω_n/K and ω/ω_n.

In a similar manner, the effective modulating signal becomes

$$v_m(\tau) = \begin{cases} \dot{\phi}(\tau), & 0 \le \tau \le \pi \\ -\dot{\phi}(\tau), & \pi \le \tau \le 2\pi \end{cases} \tag{22}$$

where

$$\dot{\phi}(\tau) = \Lambda\omega\{1 + M \exp\left(-\frac{\alpha}{\omega}\tau\right) \sin\left(\frac{\beta}{\omega}\tau + \gamma\right)\}. \tag{23}$$

The Fourier series representation of the modulating signal is

$$v_m(\tau) = \sum_{n=1,3,5,\ldots}^{\infty} |A_n| \cos\,(n\tau + \lambda_n) \tag{24}$$

due to the existance of rotational symmetry of the waveform. The complex coefficients are given by

$$A_n = \frac{1}{\pi} \int_0^{\pi} \dot{\phi}(\tau) \exp\,(-jn\tau)\, d\tau. \tag{25}$$

The complex spectrum corresponding to the effective modulating signal is therefore a function of δ, ω_n/K and ω/ω_n.

It is useful to define a new dimensionless parameter,

$$\tau_s = \frac{\text{frequency transition time, } t_s}{\text{bit length, } T/2}, \tag{26}$$

which can be conveniently related to the ratio ω/ω_n by letting $t_s = 2\pi/\omega_n$, which yields

$$\tau_s = 2\,\frac{\omega}{\omega_n}\ . \tag{27}$$

Thus, for example, $\tau_s = 0.5$ corresponds to a total frequency transition time equal to one-half of the bit length. Rectangular wave modulation thus corresponds to $\tau_s = 0$.

The spectrum corresponding to the effective modulating signal can be calculated in a straight forward manner by evaluation of (25) giving

$$|A_n| = \frac{4\ \Lambda\omega}{n\pi}\sqrt{\frac{a^2 + b^2}{c^2 + d^2}} \tag{28}$$

and

$$\lambda_n = \arctan\,(b/a) - \arctan\,(d/c),$$

where

$$a = 1$$

$$b = n(\omega/\omega_n)\ (2\delta - \omega_n/k)$$

$$c = 2n\delta\omega/\omega_n$$

$$d = 1 - (n\omega/\omega_n)^2.$$

Equation 28 yields the complex amplitude spectrum for a rectangular modulating signal by letting $\tau_s = 2\omega/\omega_n = 0$.

A means for evaluation of the integrals of (20) and (21) in closed form to yield the modulated carrier spectrum is not evident, except in the special case where $\tau_s = 0$ corresponding to rectangular wave modulation. Two approaches towards determination of the modulated carrier spectrum have been considered. The first method involves a Taylor series expansion of the instantaneous phase expression given by (14), and subsequent term by term integration to yield the desired spectral information [6]. This approach leads to excessive computation when large values of the transition ratio and/or modulation index are considered because the series converges slowly. The second approach makes use of the fast Fourier transform algorithm [7]. The fast Fourier transform technique was employed to calculate the modulated carrier spectra presented for all $\tau_s > 0$. The complex amplitude spectrum for $\tau_s = 0$ can be found as described in Appendix I.

Spectral Data

Numerous combinations of PLL parameter values are possible. In general, the spectral bandwidth of the modulating signal is more sensitive to variations of the loop parameters than the modulated carrier bandwidth. Examination of (28) indicates that the magnitude of the modulating signal amplitude spectrum drops off at a maximum rate of 12db/octave for large

n, except in the case where $\omega_n/K = 2\delta$ which results in a decrease of 18db/octave (See Figure 2). This compares with 6db/octave for $\tau_s = 0$. The PLL configuration corresponding to $\omega_n/K = 2\delta$ is relatively easy to implement since the phase lag filter shown in Figure 1 reduces to a simple RC low pass filter, allowing convenient control of the transition time. Additionally, selection of $\delta = 0.9$ provides a smooth frequency transition with small overshoot. Thus, consideration is restricted to this particular PLL configuration in the following paragraphs. Complex amplitude spectral data have been calculated which correspond to the modulating signal and the modulated carrier. These magnitude and phase data are presented in Tables 1 through 5 and Tables 6 through 30, respectively, in Appendix II. Modulated carrier spectral data are presented for modulation index values of 1, 2, 5, 10 and 25.

Spectral envelopes of the discrete spectral magnitudes are employed here to summarize the effects of modulation waveshaping upon the resulting spectra. With regards to the modulating signal, it has been shown in (24) that all even order harmonics have zero value. Thus, the odd order harmonics have discrete values lying on the envelope curve corresponding to their respective harmonic numbers. Figure 3 shows such spectral envelopes corresponding to $\delta = 0.9$ and $\omega_n/K = 1.8$ (See Figure 2) for several values of τ_s, including $\tau_s = 0$ for

comparative purposes. (These curves have been plotted from the data of Tables 1 through 5.) While the magnitude of the spectrum corresponding to $\tau_s = 0$ decreases at a rate of 6db/octave for large n, it is seen that this rate increases to a maximum of 18db/octave for relatively large values of τ_s, as indicated by (28).

Representative effects of controlled frequency transitions upon the frequency spectra of frequency shift modulated carriers are depicted in the line spectra of Figure 4. Only the sidebands above the carrier are shown since each magnitude spectrum is symmetrical about the carrier as seen from (19). Figure 4 also illustrates spectral envelopes under which all spectral lines exist. In the case of instantaneous frequency transitions ($\tau_s = 0$), certain spectral lines under the envelope are zero valued. However, controlled frequency transitions ($\tau_s > 0$) cause these lines to appear. The spectral envelopes of Figure 4 show that the spectral energy is shifted towards the carrier with increasing values of τ_s. That is, the signal bandwidth is reduced as compared with that corresponding to an instantaneous frequency transition. These effects are typical and general conclusions may be drawn.

Utilizing the data presented in Tables 6 through 30, modulated carrier spectral amplitude envelopes are shown in Figures 5 through 9 depicting dependence upon modulation index and various values of the transition ratio, including $\tau_s = 0$ curves for comparative purposes. Corresponding to $\tau_s = 0$ and

$n^2 \gg m^2$, the spectral magnitude decreases at a rate of 12db/octave. Appendix I shows that the signal bandwidth for $\tau_s = 0$ and $n^2 \gg m^2$ increases as the square root of m when compared with a modulation index of unity. For $\tau_s > 0$, the spectral magnitude drops off at a rate which increases with increasing values of modulation index.

Modulated carrier signal bandwidths as a function of the transition ratio are shown in Figure 10 for various values of modulation index. An arbitrarily selected 40db bandwidth is employed for comparative purposes. This figure shows that the maximum reduction in modulating signal bandwidth is obtained for large modulation index values; that is, the signals requiring the largest passbands. Also shown in Figure 10 is the bandwidth variation of the modulating signal with transition ratio, and it is noted that the modulating signal bandwidth is more responsive to changes in τ_s than is the modulated carrier signal bandwidth.

Conclusions

A phase lock frequency modulation technique has been investigated which has application to phase coherent FSK. This technique utilizes a phase-locked loop (PLL) to provide effective modulation wave-shaping and a corresponding reduction of the effective modulating signal and modulated signal bandwidths as compared with those resulting from rectangular wave frequency modulation, or an instantaneous

frequency shifted carrier. An analysis of the PLL response to an instantaneous frequency shift has been presented, along with representative responses as a function of the loop parameters. Spectral analyses have been conducted and results presented for this controlled frequency transition, frequency shift keyed (CFT-FSK) type modulated signal and for the corresponding effective modulating signal. A selected class of PLL parameters and various values of transition ratio and modulation index have been considered. The PLL parameters selected yield a smooth frequency transition with small overshoot of the final frequency value. The results reported here demonstrate that significant bandwidth reduction and variation of such can be realized for the modulated signal, with a more pronounced change realized for the effective modulating signal.

The analyses and results of the spectral calculations presented here suggest several areas which can benefit from this work or an appropriate extension of such. With regards to the signal bandwidth variations which can be achieved through the loop parameters, modulation frequency, modulation index and frequency transition time, a wide variety of signal designs are possible. The signal characteristics can be matched to some degree to realizable bandpass characteristics for test and evaluation purposes. Related work in the area of CFT-FSK has shown good agreement between theory and practice [8]; thus the actual signal characteristics are predictable. Such is the case because the PLL model and operational assumptions which

have been chosen are realistic ones.

Concerning the bandwidth reduction characteristics of the CFT-FSK method, no attempt has been made to optimize the PLL configuration or signal design employed here from the viewpoint of bandpass distortion and detection as applied to communications systems. This is an area which can utilize the theory and results presented here, in particular where application is made to phase coherent, narrowband, VLF-FSK systems.

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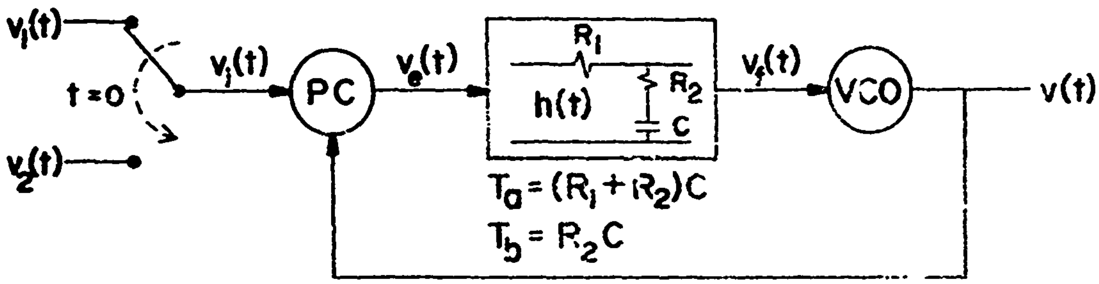

$v_1(t) = \sin(\omega_1 t)$

$v_2(t) = \sin(\omega_2 t)$

$v_i(t) = \sin[\omega_c t + \varphi_i(t)]$

$v_e(t) = k_c[\varphi_i(t) - \varphi(t)]$

$v_f(t) = \int_{-\infty}^{t} v_e(x)\, h(t-x)\, dx$

$\dot{\varphi}(t) = k_v v_f(t)$

$v(t) = \sin[\omega_c t + \varphi(t)]$

INPUT FREQUENCY, $\dot{\varphi}_i(t)$ OUTPUT FREQUENCY, $\dot{\varphi}(t)$

FIGURE I. LINEAR PLL MODEL

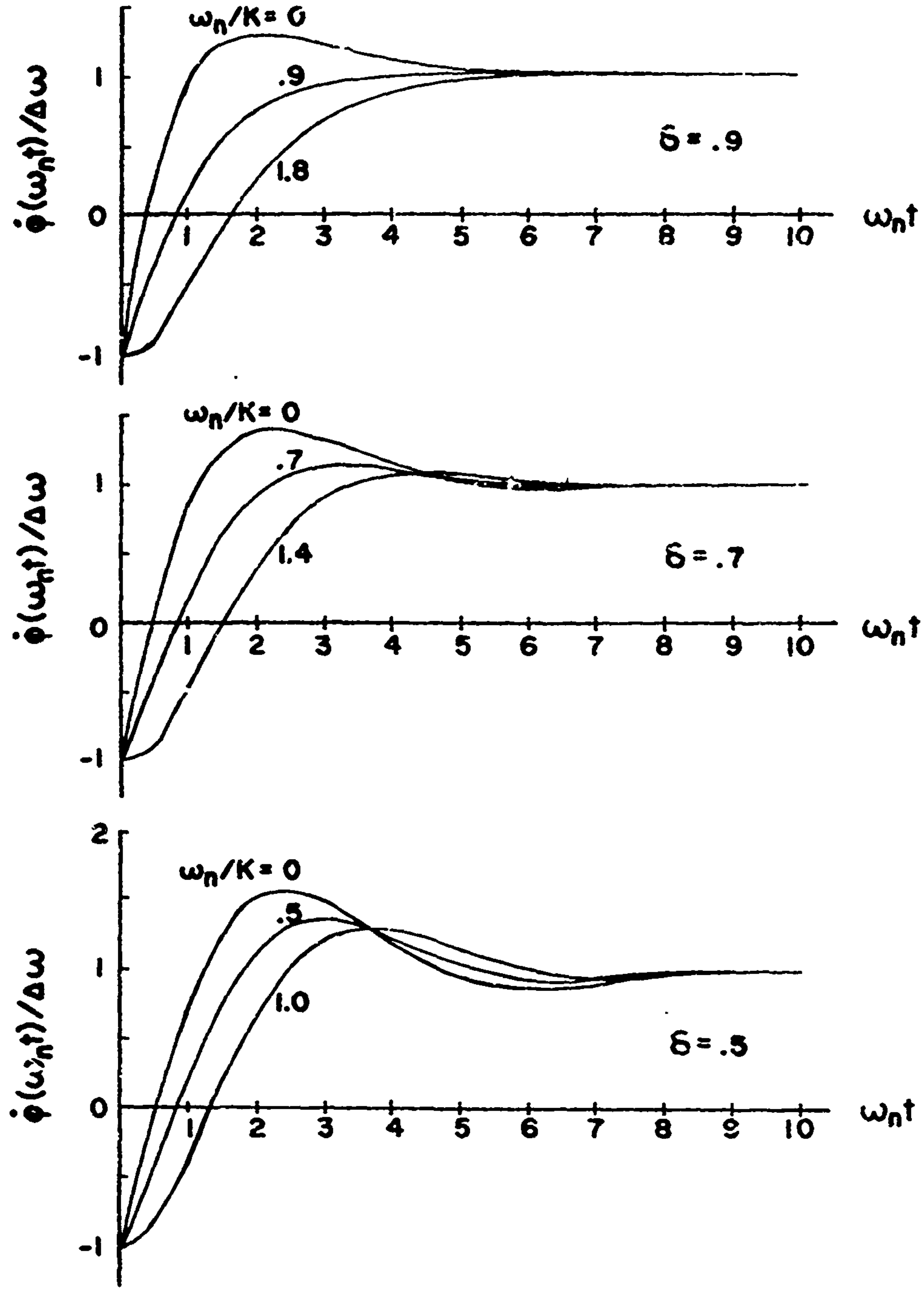

FIGURE 2. EFFECTIVE MODULATION WAVEFORMS

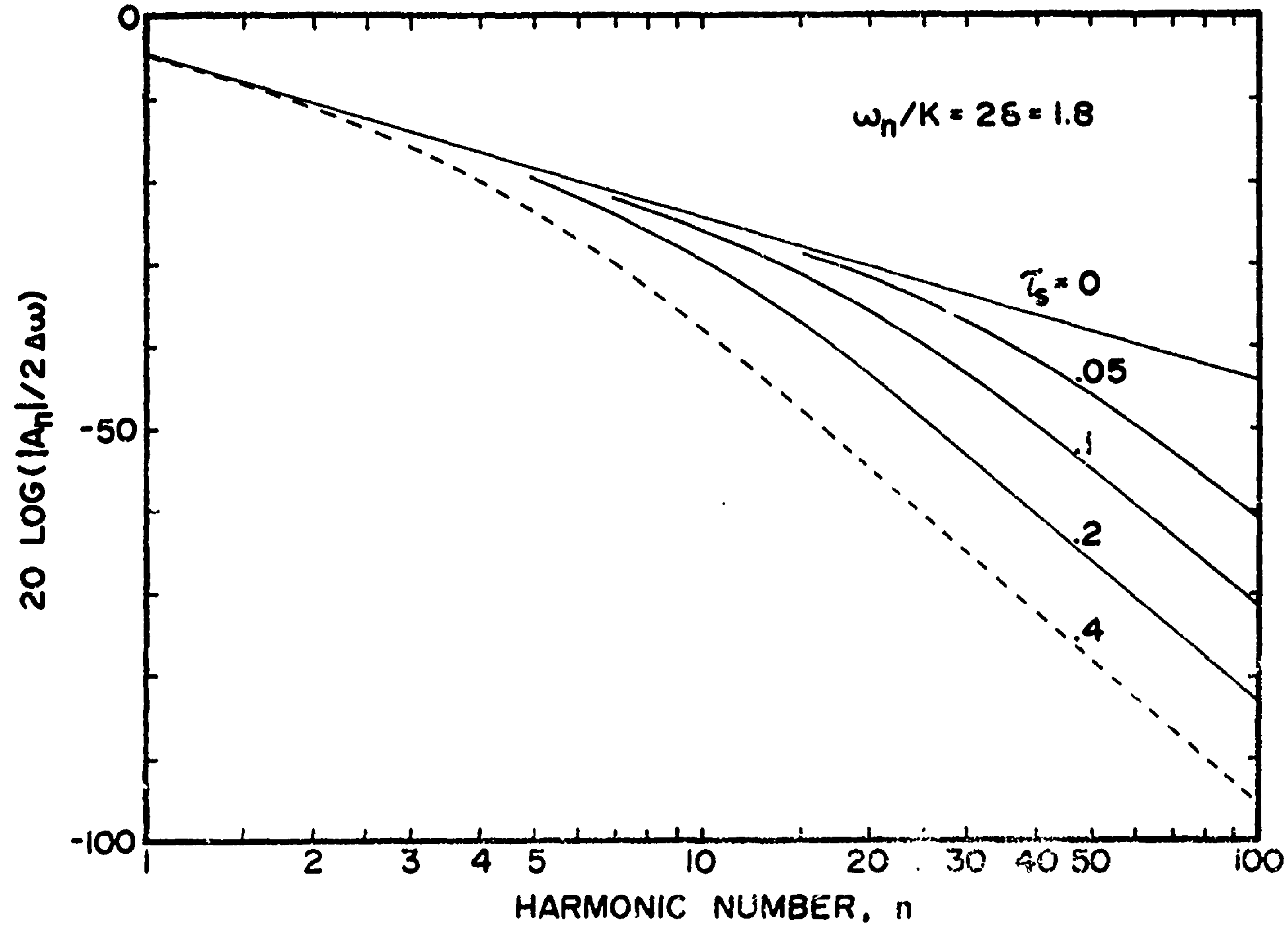

FIGURE 3. SPECTRAL ENVELOPES OF MODULATING SIGNAL

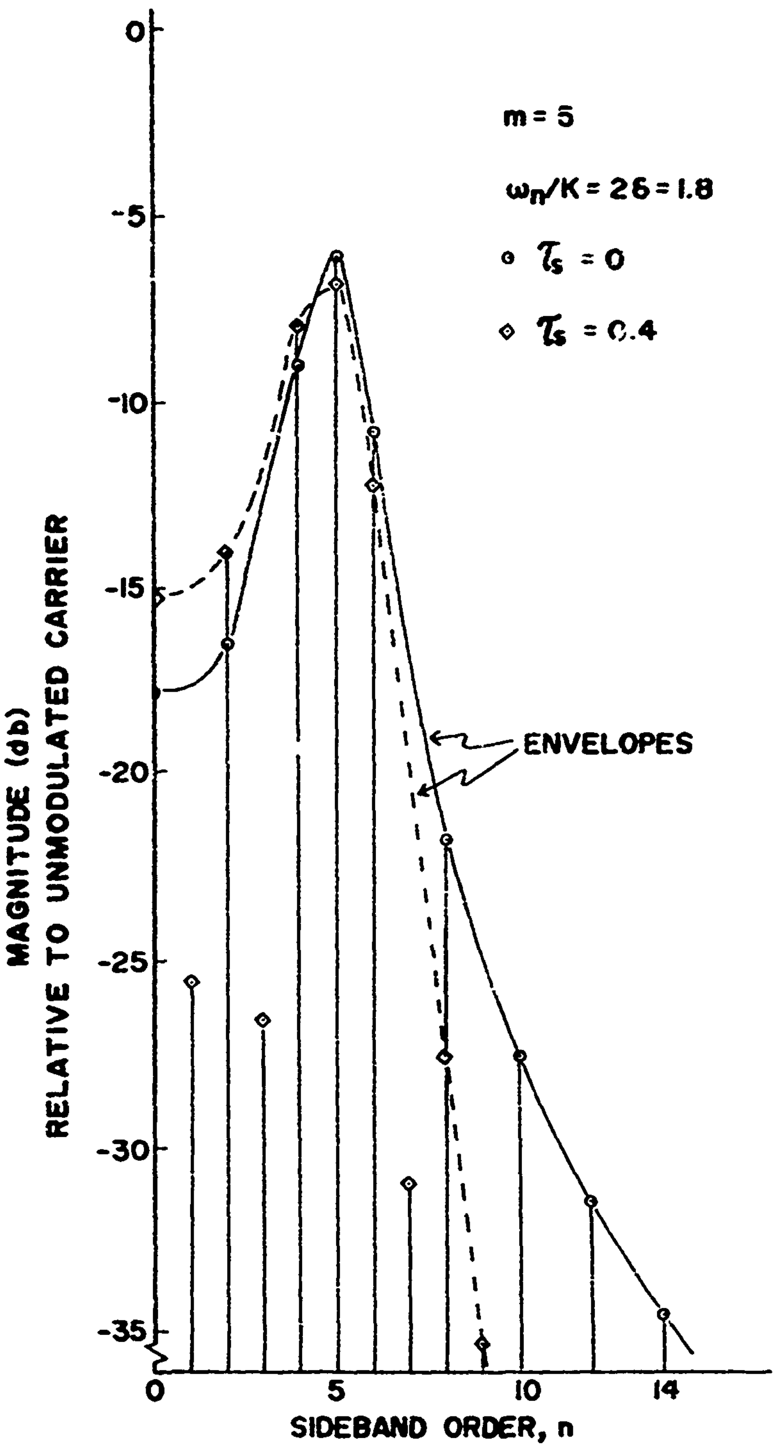

FIGURE 4. DISCRETE SPECTRA AND ASSOCIATED ENVELOPES

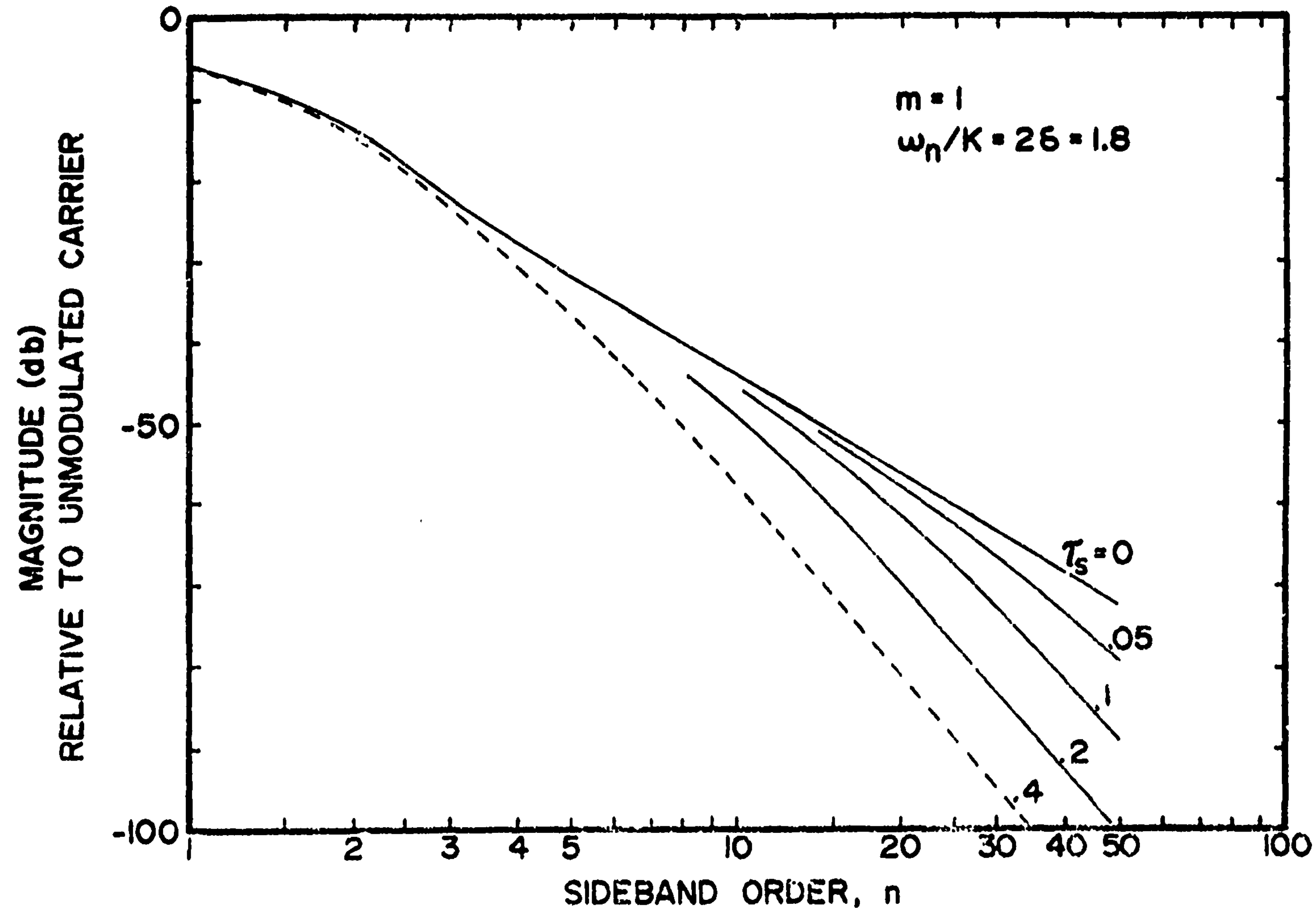

FIGURE 5. SPECTRAL ENVELOPES OF MODULATED CARRIER

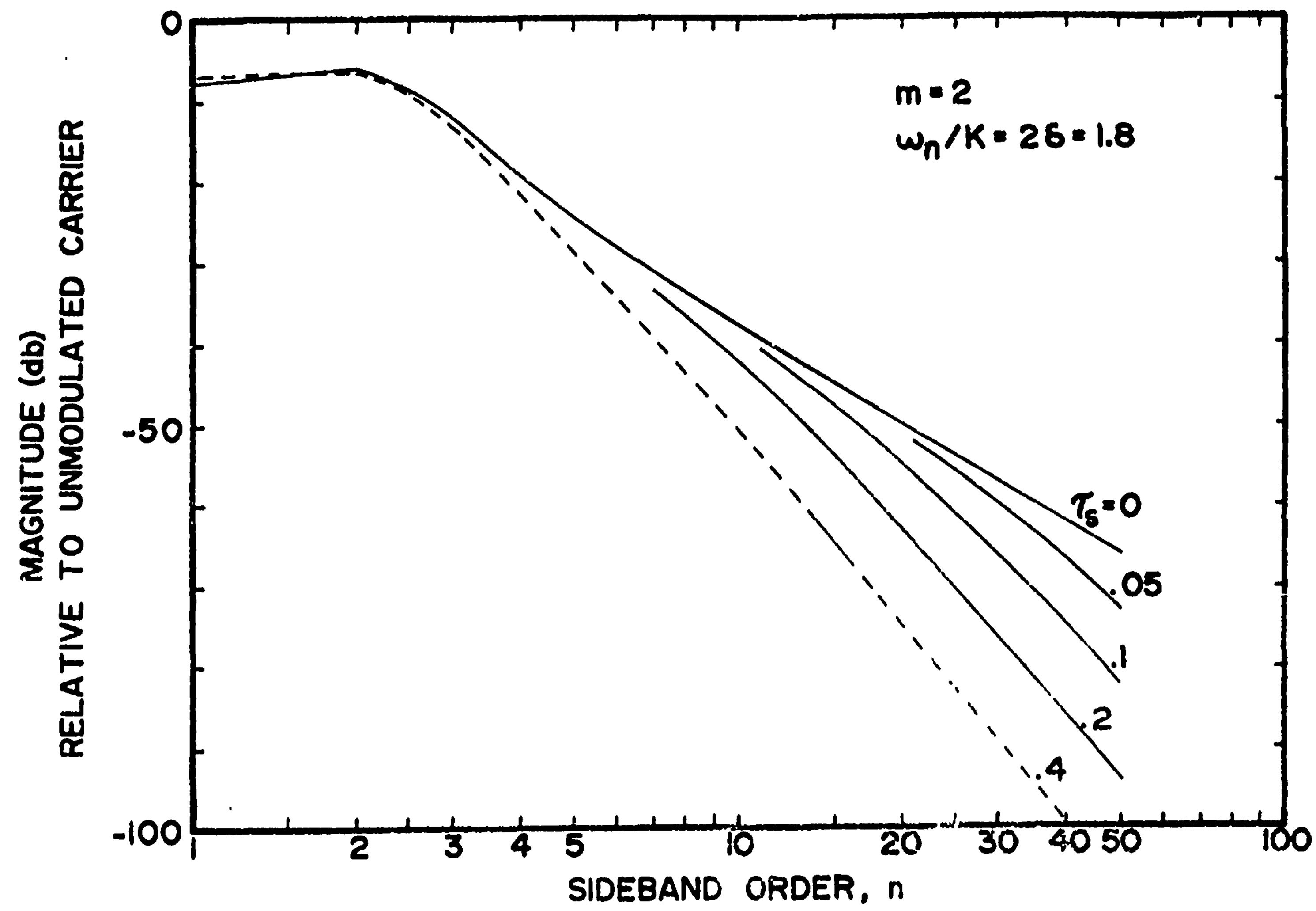

FIGURE 6. SPECTRAL ENVELOPES OF MODULATED CARRIER

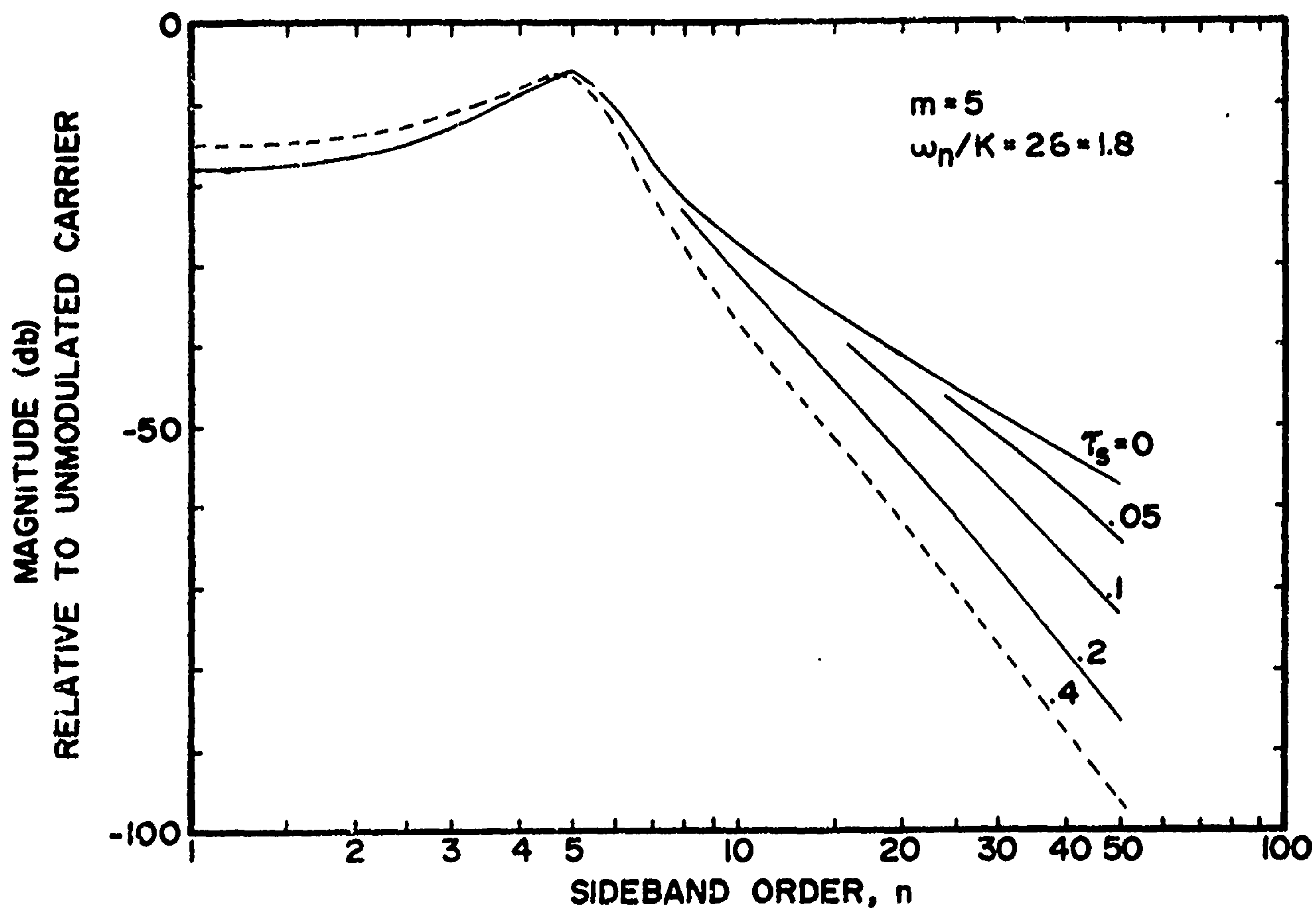

FIGURE 7. SPECTRAL ENVELOPES OF MODULATED CARRIER

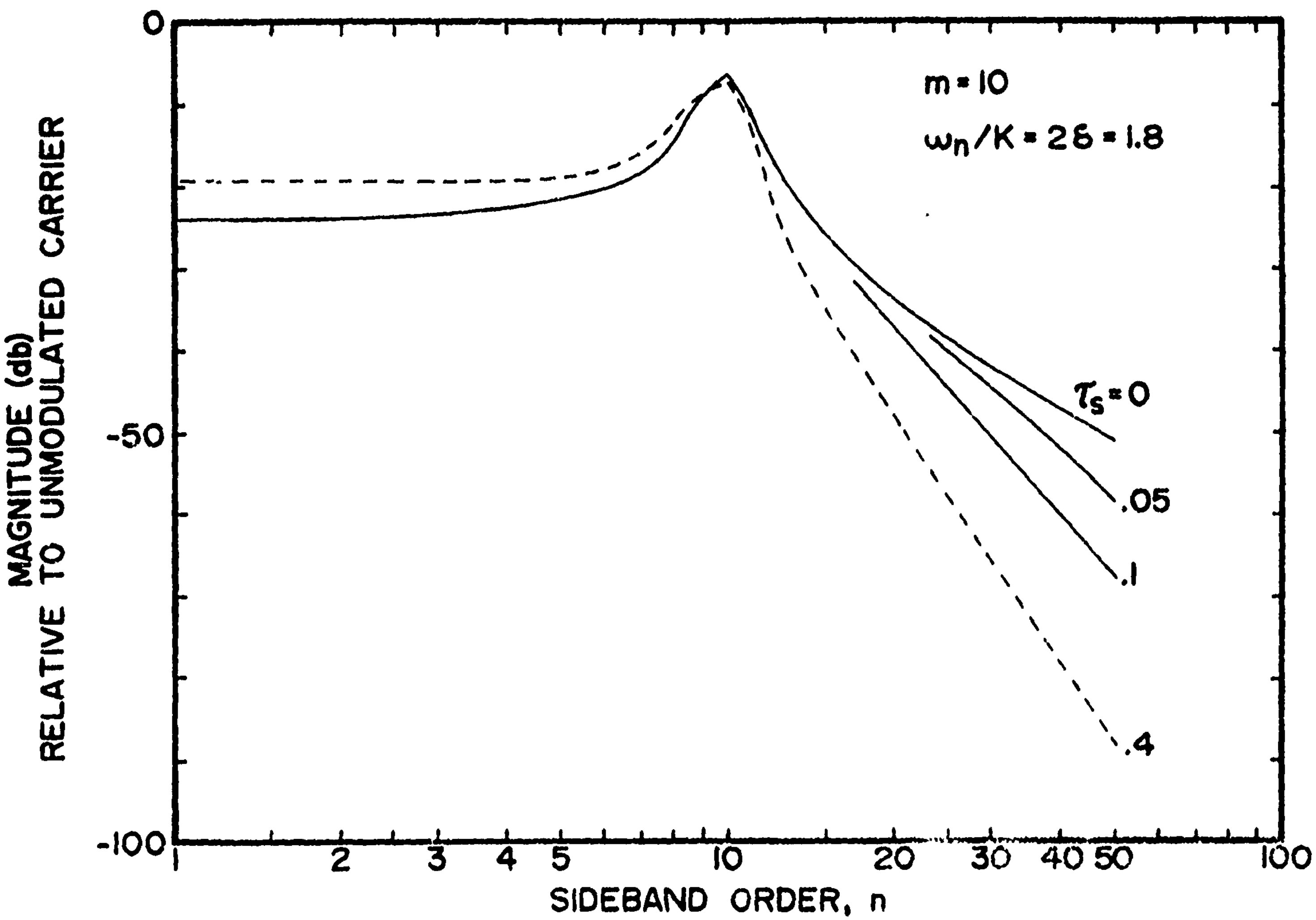

FIGURE 8. SPECTRAL ENVELOPES OF MODULATED CARRIER

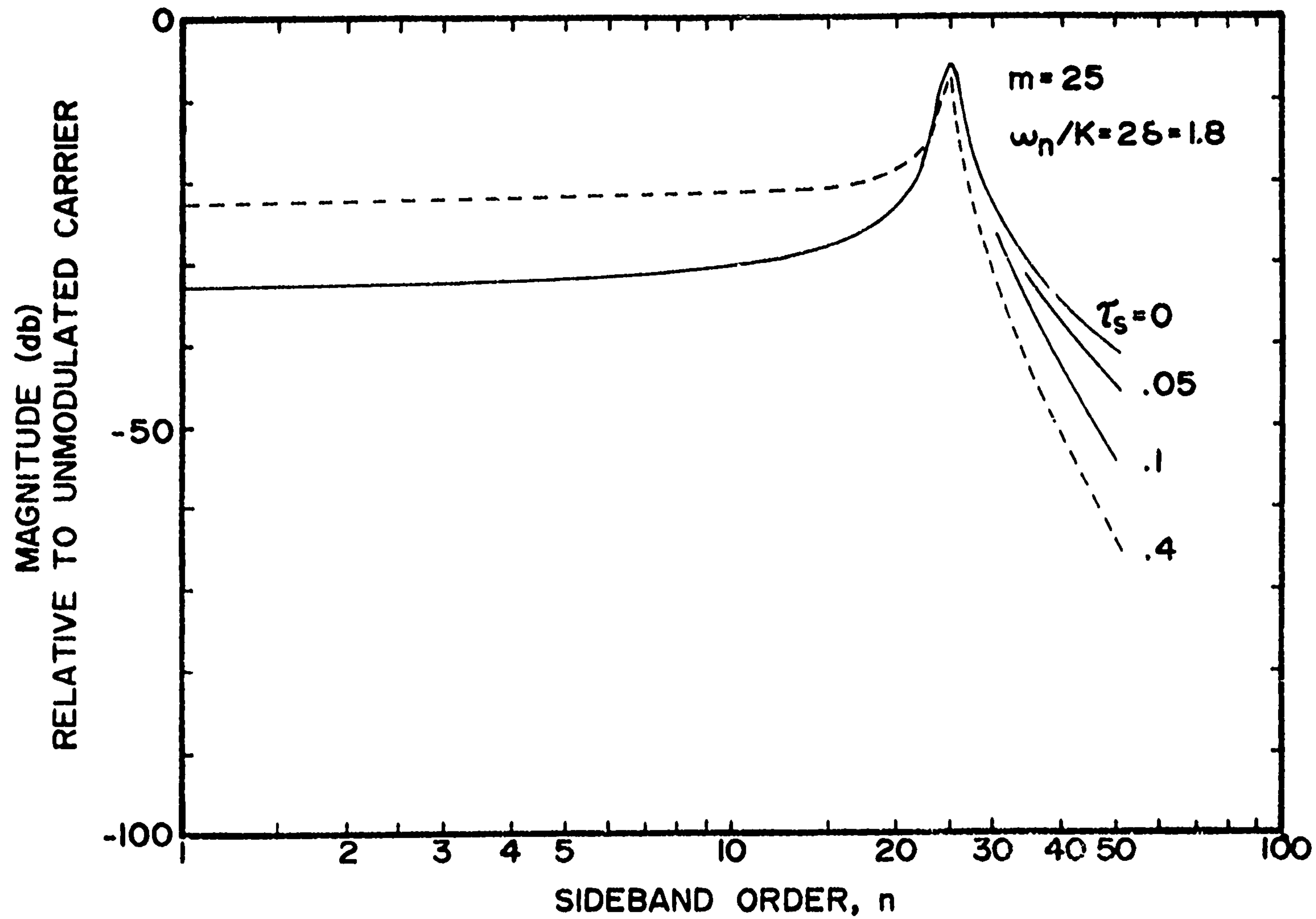

FIGURE 9. SPECTRAL ENVELOPES OF MODULATED CARRIER

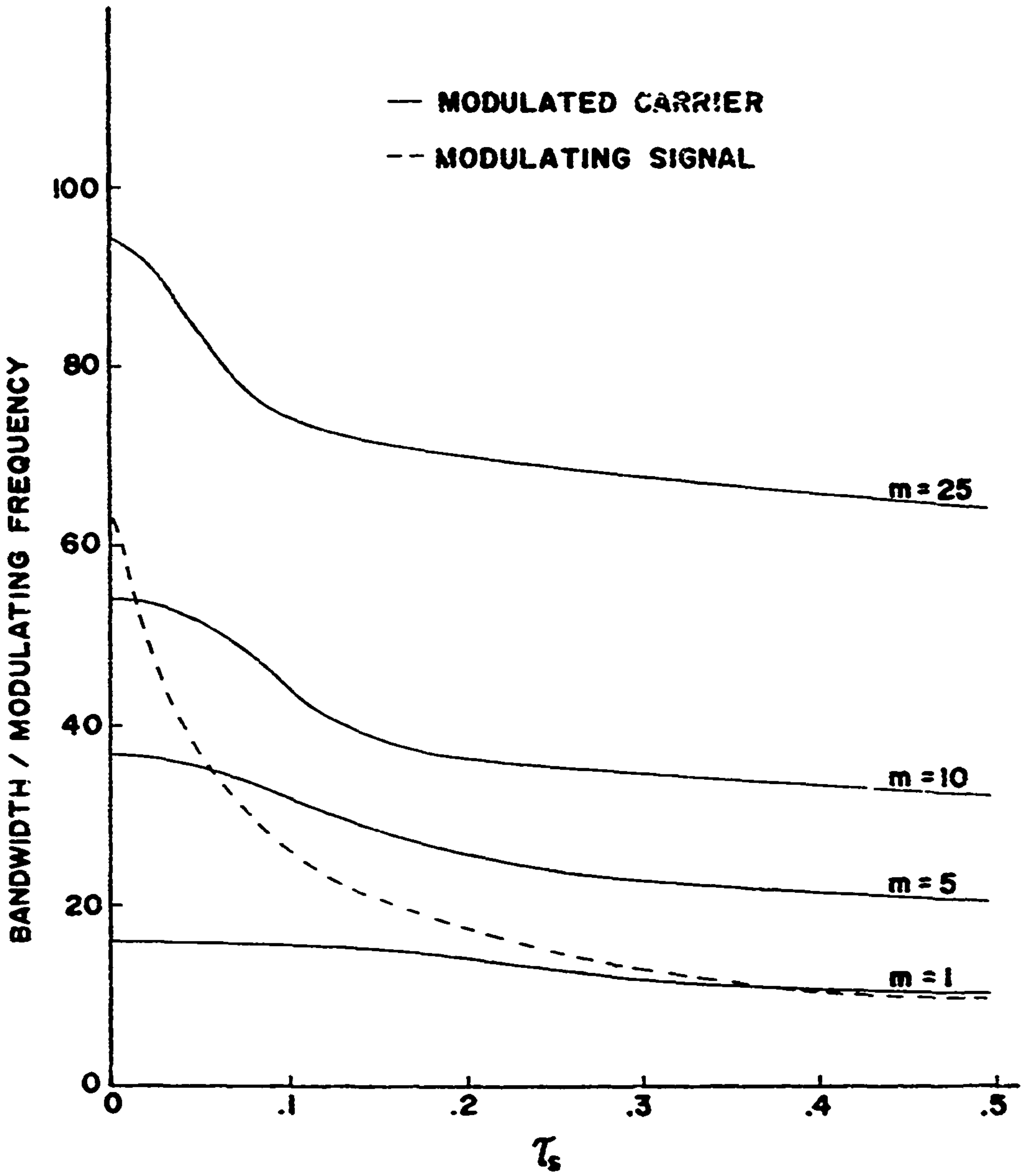

FIGURE 10. 40 db SIGNAL BANDWIDTHS

APPENDIX I

Rec‘ ˜ular Wave Modulated Carrier Spectrum

.. e phase function given by (18) for the case of instantaneous ($\tau_s = 0$), periodic frequency shifts becomes simply

$$\varphi(\tau) = m\tau.$$

Employing (20) and (21), the complex Fourier coefficients are found to be as follows.

(i) m = even integer:

$C_n = 0.5,\ n = m$

$C_n = 0$, otherwise

$$D_n = \frac{2m}{(m^2-n^2)\pi},\ n = 1, 3, 5, \ldots.$$

$D_n = 0$, otherwise

(ii) m = odd integer:

$C_n = 0.5,\ n = m$

$C_n = 0$, otherwise

$$D_n = \frac{2m}{(m^2-n^2)\pi},\ n = 0, 2, 4, \ldots.$$

$D_n = 0$, otherwise

The magnitudes of the sidebands (and spectral envelopes) for n>m are given by

$$|D_n| = \frac{2m}{(n^2-m^2)\pi}$$

Expressing the sideband magnitude in decibels,

$$|D_n|_{db} = -20 \log \left[\frac{(n^2-m^2)\pi}{2m} \right], \quad n>m.$$

An approximate expression can be written for large values of n relative to m; that is

$$|D_n|_{db} = -20 \log (\pi/2) - 40 \log (n) + 20 \log (m), \quad n^2 \gg m^2.$$

The spectral magnitude decreases at the rate 12db/octave for fixed values of m. The above expression also shows that the signal bandwidth increases as the square root of m when compared to that corresponding to unity modulation index. That is, for example, an arbitrarily defined signal bandwidth, $n^2 \gg m^2$, will increase by a factor of 5 if the modulation index is increased from 1 to 25.

APPENDIX II

Tables of Spectral Data

Tables containing spectral data are presented here for the modulating signal and the modulated carrier. These data correspond to $\omega_n/K = 2\delta = 1.8$ and $\tau_s = 0, 0.05, 0.1, 0.2$, and 0.4. With respect to the modulated carrier, data are presented for modulation index values of m = 1, 2, 5, 10 and 25.

In order to facilitate compilation and presentation of these data, the variable parameters are denoted at the head of each table as follows:

A = modulation index, m (where applicable)

B = damping factor, δ

C = ratio of loop natural undamped frequency to open loop gain, ω_n/K

D = frequency transition ratio, τ_s

N = harmonic number and sideband order, n.

Asterists (****) which appear in the heading of tables denote that the corresponding parameter is not applicable for the particular case under consideration. An example is the case of data presented for instantaneous frequency transition. Asterists (******) entries also appear in the tabulated data corresponding to amplitude spectral values whose magnitudes are more than 100db below the unmodulated carrier. Such values are deleted from the tables because computation accuracy becomes

a factor for spectral values of the order 10^{-6} and less.

Spectral data for the modulated carrier signal, with $\tau_s > 0$, was calculated using the fast Fourier transform method. Based upon known spectral values determined conventionally (Appendix I) for $\tau_s = 0$, the error introduced due to aliasing is estimated to be less than 1% for the worse case of $m = 25$ and $\tau_s = 0.05$.

TABLE 1 MODULATING SIGNAL SPECTRAL DATA

B=##### C=##### D=0.00

N	MAGNITUDE (DB)	PHASE (DEG)
1	-3.92	-90.0
3	-13.46	-90.0
5	-17.90	-90.0
7	-20.82	-90.0
9	-23.00	-90.0
11	-24.75	-90.0
13	-26.20	-90.0
15	-27.44	-90.0
17	-28.53	-90.0
19	-29.49	-90.0
21	-30.36	-90.0
23	-31.15	-90.0
25	-31.88	-90.0
27	-32.54	-90.0
29	-33.17	-90.0
31	-33.74	-90.0
33	-34.29	-90.0
35	-34.80	-90.0
37	-35.28	-90.0
39	-35.74	-90.0
41	-36.17	-90.0
43	-36.59	-90.0
45	-36.98	-90.0
47	-37.36	-90.0
49	-37.72	-90.0
51	-38.07	-90.0
53	-38.40	-90.0
55	-38.72	-90.0
57	-39.03	-90.0
59	-39.33	-90.0
61	-39.62	-90.0
63	-39.90	-90.0
65	-40.18	-90.0
67	-40.44	-90.0
69	-40.69	-90.0
71	-40.94	-90.0
73	-41.18	-90.0
75	-41.42	-90.0
77	-41.65	-90.0
79	-41.87	-90.0
81	-42.09	-90.0
83	-42.30	-90.0
85	-42.51	-90.0
87	-42.71	-90.0
89	-42.91	-90.0
91	-43.10	-90.0
93	-43.29	-90.0
95	-43.47	-90.0
97	-43.65	-90.0
99	-43.83	-90.0

TABLE 2 MODULATING SIGNAL SPECTRAL DATA

B=0.90 C=1.80 D=0.05

N	MAGNITUDE (DB)	PHASE (DEG)
1	-3.92	-92.5
3	-13.49	-97.7
5	-17.98	-102.8
7	-20.99	-108.0
9	-23.28	-113.1
11	-25.16	-118.1
13	-26.77	-123.1
15	-28.21	-128.1
17	-29.52	-133.0
19	-30.73	-137.8
21	-31.88	-142.5
23	-32.97	-147.1
25	-34.02	-151.5
27	-35.03	-155.9
29	-36.02	-160.0
31	-36.98	-164.0
33	-37.92	-167.8
35	-38.84	-171.5
37	-39.74	-175.0
39	-40.63	-178.3
41	-41.50	178.4
43	-42.35	175.4
45	-43.18	172.5
47	-44.00	169.7
49	-44.81	167.2
51	-45.60	164.7
53	-46.37	162.4
55	-47.12	160.2
57	-47.87	158.1
59	-48.59	156.1
61	-49.31	154.2
63	-50.00	152.4
65	-50.69	150.7
67	-51.36	149.0
69	-52.01	147.5
71	-52.65	146.0
73	-53.29	144.6
75	-53.90	143.3
77	-54.51	142.0
79	-55.10	140.7
81	-55.69	139.6
83	-56.26	138.4
85	-56.82	137.4
87	-57.37	136.3
89	-57.91	135.3
91	-58.44	134.4
93	-58.96	133.5
95	-59.47	132.6
97	-59.97	131.8
99	-60.47	130.9

TABLE 3 MODULATING SIGNAL SPECTRAL DATA

B=0.90 C=1.80 D=0.10

N	MAGNITUDE (DB)	PHASE (DEG)
1	-3.93	-95.1
3	-13.58	-105.4
5	-18.24	-115.6
7	-21.49	-125.6
9	-24.12	-135.4
11	-26.41	-144.8
13	-28.51	-153.7
15	-30.48	-162.0
17	-32.36	-169.7
19	-34.17	-176.7
21	-35.90	176.8
23	-37.58	171.1
25	-39.18	165.9
27	-40.73	161.3
29	-42.21	157.1
31	-43.64	153.3
33	-45.00	149.8
35	-46.31	146.7
37	-47.58	143.9
39	-48.79	141.3
41	-49.95	139.0
43	-51.07	136.8
45	-52.15	134.9
47	-53.20	133.0
49	-54.20	131.3
51	-55.17	129.8
53	-56.11	128.3
55	-57.02	127.0
57	-57.90	125.7
59	-58.76	124.5
61	-59.58	123.4
63	-60.39	122.4
65	-61.17	121.4
67	-61.93	120.5
69	-62.67	119.6
71	-63.38	118.8
73	-64.08	118.0
75	-64.77	117.3
77	-65.43	116.6
79	-66.08	115.9
81	-66.72	115.3
83	-67.34	114.7
85	-67.94	114.1
87	-68.53	113.5
89	-69.11	113.0
91	-69.68	112.5
93	-70.24	112.0
95	-70.78	111.6
97	-71.31	111.1
99	-71.84	110.7

TABLE 4 MODULATING SIGNAL SPECTRAL DATA

B=0.90 C=1.80 D=0.20

N	MAGNITUDE (DB)	PHASE (DEG)
1	-3.97	-100.3
3	-13.95	-120.6
5	-19.27	-140.1
7	-23.49	-157.9
9	-27.25	-173.3
11	-30.73	173.9
13	-33.94	163.5
15	-36.91	155.1
17	-39.64	148.2
19	-42.17	142.6
21	-44.50	137.9
23	-46.66	133.9
25	-48.67	130.6
27	-50.55	127.6
29	-52.31	125.1
31	-53.97	122.9
33	-55.53	120.9
35	-57.01	119.2
37	-58.41	117.6
39	-59.74	116.2
41	-61.01	115.0
43	-62.22	113.8
45	-63.38	112.8
47	-64.49	111.8
49	-65.56	110.9
51	-66.58	110.1
53	-67.57	109.4
55	-68.52	108.6
57	-69.44	108.0
59	-70.32	107.4
61	-71.18	106.8
63	-72.01	106.3
65	-72.82	105.8
67	-73.60	105.3
69	-74.36	104.9
71	-75.10	104.5
73	-75.82	104.1
75	-76.52	103.7
77	-77.20	103.3
79	-77.86	103.0
81	-78.51	102.7
83	-79.14	102.4
85	-79.76	102.1
87	-80.36	101.8
89	-80.95	101.5
91	-81.53	101.3
93	-82.09	101.0
95	-82.64	100.8
97	-83.18	100.6
99	-83.71	100.4

TABLE 5 MODULATING SIGNAL SPECTRAL DATA

B=0.90 C=1.80 D=0.40

N	MAGNITUDE (DB)	PHASE (DEG)
1	-4.13	-110.5
3	-15.44	-149.3
5	-23.00	-180.0
7	-29.44	159.1
9	-34.91	145.3
11	-39.58	135.8
13	-43.61	129.0
15	-47.13	124.0
17	-50.26	120.0
19	-53.06	116.9
21	-55.60	114.4
23	-57.92	112.3
25	-60.05	110.5
27	-62.03	109.0
29	-63.86	107.7
31	-65.58	106.5
33	-67.19	105.5
35	-68.71	104.7
37	-70.15	103.9
39	-71.51	103.2
41	-72.81	102.5
43	-74.04	101.9
45	-75.22	101.4
47	-76.35	100.9
49	-77.43	100.5
51	-78.46	100.1
53	-79.46	99.7
55	-80.42	99.3
57	-81.35	99.0
59	-82.25	98.7
61	-83.11	98.4
63	-83.95	98.1
65	-84.77	97.9
67	-85.55	97.6
69	-86.32	97.4
71	-87.06	97.2
73	-87.78	97.0
75	-88.49	96.8
77	-89.17	96.6
79	-89.84	96.5
81	-90.49	96.3
83	-91.12	96.2
85	-91.74	96.0
87	-92.35	95.9
89	-92.94	95.7
91	-93.52	95.6
93	-94.08	95.5
95	-94.64	95.4
97	-95.18	95.3
99	-95.71	95.2

TABLE 6 MODULATED CARRIER SPECTRAL DATA

A= 1 B=***** C=***** D=0.00

N	MAGNITUDE (DB)	PHASE (DEG)
0	-3.92	0.0
1	-6.02	0.0
2	-13.46	180.0
3	******	******
4	-27.44	180.0
5	******	******
6	-34.80	180.0
7	******	******
8	-39.90	180.0
9	******	******
10	-43.83	180.0
11	******	******
12	-47.02	180.0
13	******	******
14	-49.72	180.0
15	******	******
16	-52.05	180.0
17	******	******
18	-54.10	180.0
19	******	******
20	-55.94	180.0
21	******	******
22	-57.60	180.0
23	******	******
24	-59.11	180.0
25	******	******
26	-60.50	180.0
27	******	******
28	-61.79	180.0
29	******	******
30	-62.99	180.0
31	******	******
32	-64.11	180.0
33	******	******
34	-65.17	180.0
35	******	******
36	-66.16	180.0
37	******	******
38	-67.10	180.0
39	******	******
40	-67.99	180.0
41	******	******
42	-68.84	180.0
43	******	******
44	-69.65	180.0
45	******	******
46	-70.42	180.0
47	******	******
48	-71.16	180.0
49	******	******

TABLE 7 MODULATED CARRIER SPECTRAL DATA

A= 1 B=0.90 C=1.80 D=0.05

N	MAGNITUDE (DB)	PHASE (DEG)
0	-3.91	0.0
1	-6.02	-2.5
2	-13.47	174.8
3	******	******
4	-27.49	169.6
5	******	******
6	-34.92	164.5
7	******	******
8	-40.12	159.4
9	******	******
10	-44.17	154.3
11	******	******
12	-47.51	149.3
13	******	******
14	-50.38	144.3
15	******	******
16	-52.92	139.3
17	******	******
18	-55.21	134.5
19	******	******
20	-57.31	129.7
21	******	******
22	-59.26	125.1
23	******	******
24	-61.08	120.6
25	******	******
26	-62.81	116.2
27	******	******
28	-64.45	112.0
29	******	******
30	-66.03	107.9
31	******	******
32	-67.54	104.0
33	******	******
34	-69.00	100.2
35	******	******
36	-70.41	96.6
37	******	******
38	-71.77	93.2
39	******	******
40	-73.10	89.9
41	******	******
42	-74.38	86.8
43	******	******
44	-75.63	83.9
45	******	******
46	-76.84	81.0
47	******	******
48	-78.03	78.4
49	******	******

TABLE 8 MODULATED CARRIER SPECTRAL DATA

A= 1 B=0.90 C=1.80 D=0.10

N	MAGNITUDE (DB)	PHASE (DEG)
0	-3.90	0.0
1	-6.02	-5.1
2	-13.50	169.6
3	-87.96	163.1
4	-27.64	159.4
5	-87.78	150.6
6	-35.28	149.3
7	-87.65	137.0
8	-40.77	139.3
9	-87.65	122.2
10	-45.19	129.7
11	-87.85	106.9
12	-48.98	120.6
13	-88.27	91.5
14	-52.37	111.9
15	-88.90	76.2
16	-55.46	103.9
17	-89.72	61.4
18	-58.33	96.5
19	-90.68	47.5
20	-61.03	89.8
21	-91.77	34.3
22	-63.56	83.8
23	-92.96	21.9
24	-65.96	78.3
25	-94.20	10.5
26	-68.24	73.4
27	-95.50	0.0
28	-70.40	69.0
29	-96.82	-9.6
30	-72.46	65.0
31	-98.18	-18.9
32	-74.42	61.4
33	-99.49	-27.1
34	-76.28	58.1
35	******	******
36	-78.07	55.2
37	******	******
38	-79.77	52.5
39	******	******
40	-81.41	50.0
41	******	******
42	-82.97	47.8
43	******	******
44	-84.48	45.7
45	******	******
46	-85.93	43.8
47	******	******
48	-87.32	42.0
49	******	******

TABLE 9 MODULATED CARRIER SPECTRAL DATA

A= 1 B=0.90 C=1.80 D=0.20

N	MAGNITUDE (DB)	PHASE (DEG)
0	-3.86	0.0
1	-6.02	-10.3
2	-13.62	159.4
3	-69.66	143.7
4	-28.25	139.3
5	-69.70	114.6
6	-36.70	120.5
7	-70.52	83.9
8	-43.26	103.7
9	-72.12	54.6
10	-48.87	89.5
11	-74.28	28.2
12	-53.84	77.9
13	-76.77	5.3
14	-58.30	68.5
15	-79.41	-14.2
16	-62.33	61.0
17	-82.09	-30.9
18	-66.00	54.8
19	-84.74	-45.1
20	-69.34	49.6
21	-87.33	-57.4
22	-72.42	45.3
23	-89.82	-67.9
24	-75.26	41.6
25	-92.22	-77.0
26	-77.91	38.5
27	-94.53	-84.9
28	-80.37	35.7
29	-96.72	-91.8
30	-82.69	33.3
31	-98.76	-98.0
32	-84.86	31.1
33	******	******
34	-86.92	29.3
35	******	******
36	-88.86	27.6
37	******	******
38	-90.71	26.1
39	******	******
40	-92.46	24.7
41	******	******
42	-94.12	23.6
43	******	******
44	-95.72	22.5
45	******	******
46	-97.24	21.6
47	******	******
48	-98.72	20.5
49	******	******

TABLE 10 MODULATED CARRIER SPECTRAL DATA

A= 1 B=0.90 C=1.80 D=0.40

N	MAGNITUDE (DB)	PHASE (DEG)
0	-3.71	0.0
1	-6.06	-20.6
2	-14.10	139.2
3	-52.09	99.5
4	-30.57	102.9
5	-55.10	41.7
6	-41.47	76.4
7	-59.95	-4.4
8	-50.13	59.4
9	-65.29	-38.4
10	-57.20	48.1
11	-70.49	-63.2
12	-63.14	40.0
13	-75.31	-81.6
14	-68.28	33.8
15	-79.73	-95.5
16	-72.82	29.0
17	-83.76	-106.2
18	-76.87	25.3
19	-87.43	-114.7
20	-80.52	22.4
21	-90.80	-121.6
22	-83.84	20.1
23	-93.88	-127.1
24	-86.87	18.2
25	-96.75	-131.7
26	-89.67	16.7
27	-99.41	-135.7
28	-92.26	15.4
29	******	******
30	-94.66	14.4
31	******	******
32	-96.91	13.2
33	******	******
34	-99.01	12.6
35	******	******
36	******	******
37	******	******
38	******	******
39	******	******
40	******	******
41	******	******
42	******	******
43	******	******
44	******	******
45	******	******
46	******	******
47	******	******
48	******	******
49	******	******

TABLE 11 MODULATED CARRIER SPECTRAL DATA

A= 2 B=***** C=***** D=0.00

N	MAGNITUDE (DB)	PHASE (DEG)
0	******	******
1	-7.44	0.0
2	-6.02	0.0
3	-11.88	180.0
4	******	******
5	-24.34	180.0
6	******	******
7	-30.96	180.0
8	******	******
9	-35.63	180.0
10	******	******
11	-39.26	180.0
12	******	******
13	-42.25	180.0
14	******	******
15	-44.78	180.0
16	******	******
17	-46.99	180.0
18	******	******
19	-48.95	180.0
20	******	******
21	-50.71	180.0
22	******	******
23	-52.30	180.0
24	******	******
25	-53.76	180.0
26	******	******
27	-55.10	180.0
28	******	******
29	-56.35	180.0
30	******	******
31	-57.52	180.0
32	******	******
33	-58.61	180.0
34	******	******
35	-59.63	180.0
36	******	******
37	-60.60	180.0
38	******	******
39	-61.52	180.0
40	******	******
41	-62.39	180.0
42	******	******
43	-63.22	180.0
44	******	******
45	-64.01	180.0
46	******	******
47	-64.76	180.0
48	******	******
49	-65.49	180.0

TABLE 12 MODULATED CARRIER SPECTRAL DATA

A= 2 B=0.90 C=1.80 D=0.05

N	MAGNITUDE (DB)	PHASE (DEG)
0	-94.06	180.0
1	-7.43	-2.5
2	-6.02	-5.1
3	-11.89	172.2
4	-94.08	168.8
5	-24.41	167.1
6	-93.97	163.2
7	-31.11	161.9
8	-93.89	156.9
9	-35.89	156.8
10	-93.80	150.7
11	-39.66	151.8
12	-93.72	143.8
13	-42.81	146.8
14	-93.67	137.1
15	-45.54	141.8
16	-93.64	129.6
17	-47.97	136.9
18	-93.68	122.3
19	-50.17	132.1
20	-93.74	114.6
21	-52.21	127.4
22	-93.87	107.0
23	-54.10	122.8
24	-94.05	99.2
25	-55.88	118.4
26	-94.29	91.5
27	-57.57	114.0
28	-94.58	83.8
29	-59.18	109.9
30	-94.94	76.3
31	-60.73	105.9
32	-95.30	68.8
33	-62.22	102.0
34	-95.76	61.4
35	-63.65	98.3
36	-96.18	54.3
37	-65.04	94.8
38	-96.68	47.6
39	-66.39	91.4
40	-97.24	40.9
41	-67.70	88.3
42	-97.80	34.2
43	-68.96	85.2
44	-98.37	28.0
45	-70.20	82.3
46	-99.00	21.9
47	-71.40	79.6
48	-99.59	16.2
49	-72.56	77.0

TABLE 13 MODULATED CARRIER SPECTRAL DATA

A= 2 B=0.90 C=1.80 D=0.10

N	MAGNITUDE (DB)	PHASE (DEG)
0	-76.04	180.0
1	-7.40	-5.1
2	-6.02	-10.3
3	-11.94	164.5
4	-75.84	156.7
5	-24.62	154.3
6	-75.68	143.7
7	-31.57	144.2
8	-75.61	129.5
9	-36.67	134.4
10	-75.72	114.6
11	-40.85	125.0
12	-76.03	99.2
13	-44.48	116.1
14	-76.55	83.9
15	-47.75	107.7
16	-77.26	68.9
17	-50.75	99.9
18	-78.15	54.5
19	-53.55	92.8
20	-79.17	41.0
21	-56.19	86.4
22	-80.30	28.2
23	-58.67	80.6
24	-81.52	16.3
25	-61.02	75.4
26	-82.79	5.3
27	-63.25	70.7
28	-84.10	-4.7
29	-65.37	66.5
30	-85.43	-14.2
31	-67.39	62.7
32	-86.77	-22.8
33	-69.30	59.3
34	-88.10	-30.9
35	-71.13	56.2
36	-89.43	-38.2
37	-72.88	53.4
38	-90.77	-45.1
39	-74.55	50.8
40	-92.07	-51.5
41	-76.16	48.5
42	-93.34	-57.4
43	-77.69	46.3
44	-94.61	-62.8
45	-79.17	44.3
46	-95.84	-67.9
47	-80.59	42.5
48	-97.06	-72.6
49	-81.96	40.8

TABLE 14 MODULATED CARRIER SPECTRAL DATA

A= 2 B=0.90 C=1.80 D=0.20

N	MAGNITUDE (DB)	PHASE (DEG)
0	-57.94	180.0
1	-7.28	-10.3
2	-6.04	-20.6
3	-12.14	149.2
4	-57.67	129.1
5	-25.45	129.5
6	-58.11	99.5
7	-33.33	111.2
8	-59.28	69.6
9	-39.59	95.3
10	-61.12	41.7
11	-45.01	82.0
12	-63.42	16.9
13	-49.83	71.4
14	-65.97	-4.4
15	-54.15	62.9
16	-68.63	-22.8
17	-58.05	56.1
18	-71.31	-38.4
19	-61.58	50.6
20	-73.95	-51.8
21	-64.80	45.8
22	-76.51	-63.2
23	-67.76	41.8
24	-78.97	-73.1
25	-70.52	38.3
26	-81.33	-81.6
27	-73.08	35.2
28	-83.60	-89.0
29	-75.49	32.5
30	-85.74	-95.4
31	-77.76	30.1
32	-87.81	-101.2
33	-79.90	28.0
34	-89.77	-106.1
35	-81.92	26.1
36	-91.66	-110.6
37	-83.84	24.5
38	-93.47	-114.8
39	-85.67	23.1
40	-95.17	-118.4
41	-87.40	21.7
42	-96.80	-121.5
43	-89.05	20.6
44	-98.40	-124.4
45	-90.64	19.6
46	-99.88	-127.1
47	-92.16	18.6
48	******	******
49	-93.61	17.8

TABLE 15 MODULATED CARRIER SPECTRAL DATA

A= 2 B=0.90 C=1.80 D=0.40

N	MAGNITUDE (DB)	PHASE (DEG)
0	-39.80	180.0
1	-6.86	-20.8
2	-6.19	-41.0
3	-12.84	119.0
4	-41.57	72.0
5	-28.37	84.3
6	-45.23	19.2
7	-38.97	60.6
8	-50.29	-22.5
9	-47.28	47.1
10	-55.66	-53.0
11	-53.89	37.7
12	-60.80	-75.0
13	-59.47	29.8
14	-65.51	-91.2
15	-64.41	23.2
16	-69.80	-103.5
17	-68.87	18.0
18	-73.70	-113.2
19	-72.92	14.0
20	-77.26	-120.8
21	-76.60	11.1
22	-80.53	-127.1
23	-79.96	8.9
24	-83.55	-132.2
25	-83.05	7.3
26	-86.35	-136.4
27	-85.88	6.1
28	-88.95	-139.9
29	-88.51	5.3
30	-91.36	-142.9
31	-90.95	4.4
32	-93.64	-145.6
33	-93.22	4.0
34	-95.76	-147.7
35	-95.33	3.6
36	-97.81	-149.7
37	-97.33	3.4
38	-99.70	-151.7
39	-99.23	3.1
40	******	******
41	******	******
42	******	******
43	******	******
44	******	******
45	******	******
46	******	******
47	******	******
48	******	******
49	******	******

TABLE 16 MODULATED CARRIER SPECTRAL DATA

A= 5 B=***** C=***** D=0.00

N	MAGNITUDE (DB)	PHASE (DEG)
0	-17.90	0.0
1	******	******
2	-16.38	0.0
3	******	******
4	-9.02	0.0
5	-6.02	0.0
6	-10.77	180.0
7	******	******
8	-21.76	180.0
9	******	******
10	-27.44	180.0
11	******	******
12	-31.45	180.0
13	******	******
14	-34.60	180.0
15	******	******
16	-37.21	180.0
17	******	******
18	-39.45	180.0
19	******	******
20	-41.42	180.0
21	******	******
22	-43.17	180.0
23	******	******
24	-44.76	180.0
25	******	******
26	-46.21	180.0
27	******	******
28	-47.54	180.0
29	******	******
30	-48.78	180.0
31	******	******
32	-49.93	180.0
33	******	******
34	-51.01	180.0
35	******	******
36	-52.02	180.0
37	******	******
38	-52.98	180.0
39	******	******
40	-53.88	180.0
41	******	******
42	-54.74	180.0
43	******	******
44	-55.56	180.0
45	******	******
46	-56.35	180.0
47	******	******
48	-57.09	180.0
49	******	******

TABLE 17 MODULATED CARRIER SPECTRAL DATA

A= 5 B=0.90 C=1.80 D=0.05

N	MAGNITUDE (DB)	PHASE (DEG)
0	-17.81	0.0
1	-78.17	177.1
2	-16.31	-5.1
3	-78.13	171.5
4	-8.99	-10.3
5	-6.02	-12.8
6	-10.80	164.5
7	-78.03	159.7
8	-21.89	159.4
9	-77.94	153.5
10	-27.69	154.3
11	-77.87	147.0
12	-31.85	149.2
13	-77.81	140.2
14	-35.18	144.2
15	-77.78	133.1
16	-37.99	139.3
17	-77.79	125.8
18	-40.46	134.4
19	-77.85	118.3
20	-42.69	129.6
21	-77.95	110.8
22	-44.73	125.0
23	-78.10	103.1
24	-46.62	120.4
25	-78.31	95.5
26	-48.41	116.0
27	-78.57	87.8
28	-50.09	111.7
29	-78.87	80.2
30	-51.70	107.5
31	-79.23	72.8
32	-53.24	103.6
33	-79.62	65.4
34	-54.73	99.7
35	-80.06	58.2
36	-56.16	96.1
37	-80.54	51.2
38	-57.54	92.6
39	-81.05	44.4
40	-58.89	89.3
41	-81.59	37.8
42	-60.19	86.1
43	-82.15	31.4
44	-61.45	83.1
45	-82.74	25.3
46	-62.68	80.3
47	-83.35	19.4
48	-63.88	77.6
49	-83.97	13.6

TABLE 18 MODULATED CARRIER SPECTRAL DATA

A= 5 B=0.90 C=1.80 D=0.10

N	MAGNITUDE (DB)	PHASE (DEG)
0	-17.57	0.0
1	-60.05	174.0
2	-16.11	-10.3
3	-59.99	162.0
4	-8.91	-20.6
5	-6.03	-25.7
6	-10.91	149.1
7	-59.88	135.9
8	-22.27	139.1
9	-59.96	121.7
10	-28.41	129.3
11	-60.18	107.1
12	-32.99	119.9
13	-60.57	92.2
14	-36.81	110.9
15	-61.14	77.4
16	-40.18	102.4
17	-61.89	62.9
18	-43.25	94.6
19	-62.78	49.0
20	-46.11	87.4
21	-63.81	35.8
22	-48.78	81.0
23	-64.94	23.3
24	-51.31	75.3
25	-66.15	11.6
26	-53.69	70.2
27	-67.42	0.8
28	-55.95	65.7
29	-68.72	-9.1
30	-58.09	61.7
31	-70.05	-18.4
32	-60.11	58.2
33	-71.40	-26.9
34	-62.02	55.0
35	-72.74	-34.8
36	-63.84	52.1
37	-74.07	-42.1
38	-65.57	49.4
39	-75.40	-48.9
40	-67.23	47.0
41	-76.70	-55.1
42	-68.81	44.7
43	-77.99	-60.8
44	-70.32	42.6
45	-79.25	-66.2
46	-71.78	40.6
47	-80.49	-71.1
48	-73.18	38.7
49	-81.70	-75.7

TABLE 19 MODULATED CARRIER SPECTRAL DATA

A= 5 B=0.90 C=1.80 D=0.20

N	MAGNITUDE (DB)	PHASE (DEG)
0	-16.75	0.0
1	-42.00	166.9
2	-15.42	-20.9
3	-42.22	140.6
4	-8.62	-41.5
5	-6.14	-51.3
6	-11.26	118.5
7	-43.43	87.1
8	-23.53	99.5
9	-44.61	59.9
10	-30.88	82.2
11	-46.30	33.5
12	-36.89	67.5
13	-48.44	9.1
14	-42.18	56.4
15	-50.92	-12.5
16	-46.84	48.3
17	-53.57	-31.1
18	-50.91	42.5
19	-56.28	-46.8
20	-54.48	37.7
21	-58.97	-60.2
22	-57.66	33.3
23	-61.58	-71.4
24	-60.57	29.0
25	-64.09	-81.0
26	-63.28	24.9
27	-66.48	-89.2
28	-65.84	21.0
29	-68.76	-96.3
30	-68.26	17.5
31	-70.93	-102.5
32	-70.61	14.2
33	-72.99	-107.9
34	-72.84	11.4
35	-74.96	-112.8
36	-74.97	8.8
37	-76.84	-117.1
38	-77.01	6.7
39	-78.64	-121.0
40	-78.97	4.8
41	-80.36	-124.4
42	-80.84	3.1
43	-82.02	-127.6
44	-82.62	1.8
45	-83.60	-130.4
46	-84.35	0.7
47	-85.13	-133.0
48	-86.00	-0.2
49	-86.59	-135.3

TABLE 20 MODULATED CARRIER SPECTRAL DATA

A= 5 B=0.90 C=1.80 D=0.40

N	MAGNITUDE (DB)	PHASE (DEG)
0	-15.12	0.0
1	-25.42	154.2
2	-13.98	-41.0
3	-26.52	104.0
4	-7.93	-82.5
5	-6.66	-101.4
6	-12.22	56.6
7	-31.02	7.8
8	-27.52	21.5
9	-35.31	-37.9
10	-38.34	3.7
11	-40.82	-73.5
12	-45.65	-2.3
13	-46.45	-97.8
14	-50.87	-12.0
15	-51.58	-114.1
15	-55.58	-24.2
17	-56.12	-126.0
18	-60.07	-35.9
19	-60.19	-135.5
20	-64.31	-46.1
21	-63.92	-143.2
22	-68.28	-54.8
23	-67.38	-149.5
24	-72.00	-62.4
25	-70.59	-154.6
26	-75.48	-68.9
27	-73.59	-158.7
28	-78.73	-74.4
29	-76.37	-162.0
30	-81.79	-79.3
31	-78.98	-164.6
32	-84.66	-83.6
33	-81.41	-166.6
34	-87.35	-87.3
35	-83.69	-168.2
36	-89.89	-90.6
37	-85.82	-169.5
38	-92.28	-93.5
39	-87.84	-170.6
40	-94.53	-96.3
41	-89.75	-171.5
42	-96.63	-98.4
43	-91.54	-172.3
44	-98.75	-100.5
45	-93.21	-172.8
46	******	******
47	-94.86	-173.3
48	******	******
49	-96.37	-173.7

TABLE 21 MODULATED CARRIER SPECTRAL DATA

A=10 B=***** C=***** D=0.00

N	MAGNITUDE (DB)	PHASE (DEG)
0	******	******
1	-23.83	0.0
2	******	******
3	-23.10	0.0
4	******	******
5	-21.42	0.C
6	******	******
7	-18.07	0.0
8	******	******
9	-9.49	0.0
10	-6.02	0.0
11	-10.36	180.0
12	******	******
13	-20.69	180.0
14	******	******
15	-25.86	180.0
16	******	******
17	-29.45	180.0
18	******	******
19	-32.25	180.0
20	******	******
21	-34.57	180.0
22	******	******
23	-36.57	180.0
24	******	******
25	-38.32	180.0
26	******	******
27	-39.89	180.0
28	******	******
29	-41.31	180.0
30	******	******
31	-42.62	180.0
32	******	******
33	-43.82	180.0
34	******	******
35	-44.94	180.0
36	******	******
37	-45.99	180.0
38	******	******
39	-46.97	180.0
40	******	******
41	-47.90	180.0
42	******	******
43	-48.77	180.0
44	******	******
45	-49.61	180.0
46	******	******
47	-50.40	180.0
48	******	******
49	-51.16	180.0

TABLE 22 MODULATED CARRIER SPECTRAL DATA

A=10 B=0.90 C=1.80 D=0.05

N	MAGNITUDE (DB)	PHASE (DEG)
0	-66.07	180.0
1	-23.51	-2.5
2	-66.06	174.0
3	-22.80	-7.7
4	-66.04	168.1
5	-21.18	-12.9
6	-66.00	162.0
7	-17.90	-18.0
8	-65.96	155.8
9	-9.43	-23.2
10	-6.02	-25.7
11	-10.43	151.6
12	-65.91	142.7
13	-20.92	146.6
14	-65.90	135.9
15	-26.26	141.6
16	-65.92	128.9
17	-30.06	136.6
18	-65.98	121.7
19	-33.10	131.7
20	-66.07	114.4
21	-35.68	126.9
22	-66.20	107.1
23	-37.96	122.2
24	-66.37	99.6
25	-40.02	117.6
26	-66.59	92.2
27	-41.93	113.1
28	-66.86	84.8
29	-43.71	108.7
30	-67.16	77.4
31	-45.39	104.5
32	-67.52	70.1
33	-46.99	100.4
34	-67.91	63.0
35	-48.53	96.5
36	-68.34	55.9
37	-50.01	92.7
38	-68.80	49.0
39	-51.43	89.1
40	-69.30	42.3
41	-52.81	85.8
42	-69.83	35.8
43	-54.15	82.5
44	-70.38	29.4
45	-55.45	79.5
46	-70.96	23.3
47	-56.71	76.6
48	-71.56	17.4
49	-57.94	73.9

TABLE 23 MODULATED CARRIER SPECTRAL DATA

A=10 B=0.90 C=1.80 D=0.10

N	MAGNITUDE (DB)	PHASE (DEG)
0	-47.99	180.0
1	-22.70	-5.2
2	-48.02	166.9
3	-22.06	-15.7
4	-48.10	153.8
5	-20.57	-26.1
6	-48.24	140.6
7	-17.49	-36.4
8	-48.43	127.4
9	-9.28	-46.5
10	-6.08	-51.4
11	-10.60	123.4
12	-49.03	100.6
13	-21.47	113.6
14	-49.45	87.1
15	-27.27	104.1
16	-49.98	73.4
17	-31.60	95.0
18	-50.63	59.9
19	-35.24	86.3
20	-51.41	46.5
21	-38.49	78.2
22	-52.32	33.4
23	-41.49	70.9
24	-53.34	20.9
25	-44.29	64.4
26	-54.46	9.1
27	-46.93	58.8
28	-55.67	-2.0
29	-49.42	54.1
30	-56.94	-12.5
31	-51.75	50.1
32	-58.25	-22.1
33	-53.93	46.7
34	-59.59	-31.1
35	-55.97	43.8
36	-60.95	-39.3
37	-57.87	41.2
38	-62.30	-46.8
39	-59.65	38.8
40	-63.65	-53.8
41	-61.33	36.6
42	-64.99	-60.2
43	-62.91	34.4
44	-66.31	-66.0
45	-64.43	32.2
46	-67.60	-71.4
47	-65.88	30.1
48	-68.87	-76.4
49	-67.28	28.0

TABLE 24 MODULATED CARRIER SPECTRAL DATA

A=10 B=0.90 C=1.80 D=0.20

N	MAGNITUDE (DB)	PHASE (DEG)
0	-31.29	180.0
1	-21.08	-10.2
2	-31.44	154.2
3	-20.52	-30.7
4	-31.86	128.8
5	-19.24	-51.4
6	-32.54	104.0
7	-16.55	-72.2
8	-33.40	79.9
9	-8.91	-92.8
10	-6.33	-102.3
11	-11.04	66.7
12	-35.55	32.2
13	-23.04	46.9
14	-37.04	7.8
15	-30.37	29.1
16	-38.97	-15.9
17	-36.53	15.1
18	-41.34	-37.9
19	-41.96	6.2
20	-44.01	-57.2
21	-46.52	1.9
22	-46.84	-73.5
23	-50.14	-0.7
24	-49.69	-86.9
25	-53.08	-4.3
26	-52.47	-97.8
27	-55.66	-9.2
28	-55.11	-106.7
29	-58.09	-15.0
30	-57.60	-114.1
31	-60.45	-21.1
32	-59.94	-120.5
33	-62.75	-27.2
34	-62.14	-126.0
35	-64.99	-33.1
36	-64.23	-131.0
37	-67.17	-38.6
38	-66.22	-135.5
39	-69.29	-43.6
40	-68.11	-139.6
41	-71.35	-48.4
42	-69.94	-143.2
43	-73.33	-52.8
44	-71.70	-146.5
45	-75.25	-56.8
46	-73.40	-149.5
47	-77.11	-60.6
48	-75.03	-152.2
49	-78.91	-64.1

TABLE 25 MODULATED CARRIER SPECTRAL DATA

A=10 B=0.90 C=1.80 D=0.40

N	MAGNITUDE (DB)	PHASE (DEG)
0	-19.03	180.0
1	-23.65	-15.9
2	-19.06	139.1
3	-22.08	-50.4
4	-19.23	96.5
5	-19.43	-90.9
6	-19.87	51.1
7	-15.85	-136.6
8	-21.36	4.4
9	-8.40	176.5
10	-7.21	156.0
11	-11.92	-48.5
12	-26.50	-89.7
13	-27.36	-84.8
14	-31.35	-135.8
15	-38.11	-90.2
16	-37.55	-169.9
17	-42.81	-89.7
18	-43.63	169.6
19	-46.43	-103.0
20	-48.67	157.1
21	-50.31	-117.3
22	-52.86	146.3
23	-54.13	-129.3
24	-56.63	136.0
25	-57.72	-139.1
26	-60.18	126.4
27	-61.07	-147.4
28	-63.54	117.9
29	-64.23	-154.6
30	-66.73	110.3
31	-67.21	-160.6
32	-69.76	103.7
33	-70.03	-165.6
34	-72.62	97.9
35	-72.70	-169.8
36	-75.34	92.9
37	-75.22	-173.3
38	-77.92	88.5
39	-77.60	-176.1
40	-80.37	84.6
41	-79.86	-178.3
42	-82.68	81.2
43	-81.99	179.8
44	-84.88	78.2
45	-84.01	178.4
46	-86.95	75.5
47	-85.91	177.2
48	-88.94	73.1
49	-87.73	176.4

TABLE 26 MODULATED CARRIER SPECTRAL DATA

A=25 B=***** C=***** D=0.00

N	MAGNITUDE (DB)	PHASE (DEG)
0	-31.88	0.0
1	******	******
2	-31.82	0.0
3	******	******
4	-31.65	0.0
5	******	******
6	-31.36	0.0
7	******	******
8	-30.94	0.0
9	******	******
10	-30.36	0.0
11	******	******
12	-29.60	0.0
13	******	******
14	-28.61	0.0
15	******	******
16	-27.30	0.0
17	******	******
18	-25.53	0.0
19	******	******
20	-23.00	0.0
21	******	******
22	-18.94	0.0
23	******	******
24	-9.76	0.0
[illegible]	-6.02	0.0
[illegible]	-10.11	180.0
27	******	******
[illegible]	-19.99	180.0
29	******	******
30	-24.75	180.0
31	******	******
32	-27.98	180.0
33	******	******
34	-30.46	180.0
35	******	******
36	-32.49	180.0
37	******	******
38	-34.22	180.0
39	******	******
40	-35.74	180.0
41	******	******
42	-37.09	180.0
43	******	******
44	-38.31	180.0
45	******	******
46	-39.43	180.0
47	******	******
48	-40.46	180.0
49	******	******

TABLE 27 MODULATED CARRIER SPECTRAL DATA

A=25 B=0.90 C=1.80 D=0.05

N	MAGNITUDE (DB)	PHASE (DEG)
0	-30.25	0.0
1	-50.28	176.6
2	-30.21	-5.2
3	-50.31	169.9
4	-30.07	-10.5
5	-50.35	163.2
6	-29.83	-15.8
7	-50.41	156.5
8	-29.48	-21.0
9	-50.49	149.9
10	-29.00	-26.2
11	-50.59	143.3
12	-28.36	-31.4
13	-50.71	136.8
14	-27.50	-36.6
15	-50.84	130.3
16	-26.35	-41.7
17	-50.98	123.8
18	-24.76	-46.8
19	-51.14	117.3
20	-22.43	-51.9
21	-51.30	110.9
22	-18.58	-56.9
23	-51.48	104.4
24	-9.64	-61.9
25	-6.06	-64.3
26	-10.24	113.0
27	-51.89	91.5
28	-20.40	108.1
29	-52.11	85.0
30	-25.46	103.2
31	-52.36	78.5
32	-29.02	98.4
33	-52.62	72.0
34	-31.85	93.6
35	-52.91	65.4
36	-34.26	88.9
37	-53.22	58.9
38	-36.39	84.3
39	-53.56	52.3
40	-38.34	79.8
41	-53.93	45.8
42	-40.14	75.5
43	-54.33	39.4
44	-41.85	71.3
45	-54.75	33.0
46	-43.47	67.3
47	-55.21	26.6
48	-45.03	63.5
49	-55.70	20.4

TABLE 28 MODULATED CARRIER SPECTRAL DATA

A=25 B=0.90 C=1.80 D=0.10

N	MAGNITUDE (DB)	PHASE (DEG)
0	-28.83	0.0
1	-34.69	173.9
2	-28.78	-9.7
3	-34.74	161.9
4	-28.63	-19.5
5	-34.82	149.8
6	-28.38	-29.4
7	-34.96	137.7
8	-28.03	-39.4
9	-35.14	125.6
10	-27.56	-49.6
11	-35.39	113.4
12	-26.95	-60.0
13	-35.69	101.4
14	-26.18	-70.5
15	-36.05	89.4
16	-25.15	-81.1
17	-36.46	77.6
18	-23.73	-91.8
19	-36.92	65.9
20	-21.62	-102.4
21	-37.40	54.3
22	-18.05	-113.0
23	-37.91	42.8
24	-9.44	-123.6
25	-6.21	-128.4
26	-10.45	45.8
27	-39.04	19.6
28	-21.09	35.3
29	-39.69	7.7
30	-26.73	25.1
31	-40.42	-4.2
32	-30.98	15.3
33	-41.25	-16.2
34	-34.62	6.1
35	-42.20	-28.1
36	-37.93	-2.0
37	-43.27	-39.7
38	-41.04	-9.0
39	-44.45	-50.8
40	-43.98	-14.4
41	-45.73	-61.3
42	-46.72	-18.2
43	-47.09	-71.1
44	-49.22	-20.5
45	-48.51	-80.2
46	-51.43	-21.6
47	-49.97	-88.4
48	-53.33	-22.3
49	-51.45	-95.9

TABLE 29 MODULATED CARRIER SPECTRAL DATA

A=25 B=0.90 C=1.80 D=0.20

N	MAGNITUDE (DB)	PHASE (DEG)
0	-44.34	0.0
1	-24.72	170.5
2	-43.32	-3.9
3	-24.70	151.5
4	-41.00	-14.7
5	-24.64	132.3
6	-38.33	-31.7
7	-24.58	112.7
8	-35.69	-52.1
9	-24.52	92.4
10	-33.16	-74.4
11	-24.50	71.4
12	-30.75	-97.9
13	-24.55	49.5
14	-28.43	-122.2
15	-24.72	26.8
16	-26.16	-147.1
17	-25.09	3.2
18	-23.85	-172.5
19	-25.69	-20.7
20	-21.25	162.2
21	-26.53	-44.5
22	-17.58	137.6
23	-27.52	-67.8
24	-9.20	113.8
25	-6.66	103.2
26	-10.81	-89.3
27	-29.90	-115.7
28	-22.60	-111.3
29	-31.57	-140.7
30	-30.12	-129.9
31	-33.76	-165.3
32	-36.82	-140.3
33	-36.47	172.1
34	-42.38	-138.3
35	-39.56	153.1
36	-45.52	-131.2
37	-42.81	138.2
38	-47.18	-131.1
39	-45.98	127.3
40	-48.70	-136.8
41	-48.87	119.5
42	-50.43	-144.6
43	-51.41	113.7
44	-52.32	-152.5
45	-53.62	108.5
46	-54.28	-159.9
47	-55.61	103.3
48	-56.23	-166.5
49	-57.46	98.0

TABLE 30 MODULATED CARRIER SPECTRAL DATA

A=25 B=0.90 C=1.80 D=0.40

N	MAGNITUDE (DB)	PHASE (DEG)
0	-22.58	180.0
1	-27.50	-19.2
2	-22.46	144.6
3	-28.21	-58.3
4	-22.15	108.7
5	-29.84	-99.5
6	-21.72	72.1
7	-33.09	-147.3
8	-21.30	34.3
9	-38.69	132.8
10	-21.05	-5.1
11	-34.22	17.5
12	-21.16	-46.6
13	-27.59	-43.2
14	-21.96	-91.2
15	-23.29	-92.8
16	-24.09	-141.5
17	-20.39	-141.6
18	-29.20	148.4
19	-18.58	167.9
20	-28.37	5.4
21	-18.00	115.4
22	-18.93	-74.2
23	-18.81	63.8
24	-9.03	-130.3
25	-7.83	-153.4
26	-11.67	-0.2
27	-23.01	-37.8
28	-28.59	-35.9
29	-28.22	-87.4
30	-37.67	-5.3
31	-35.39	-118.5
32	-38.78	-16.5
33	-41.44	-126.7
34	-42.32	-36.6
35	-44.98	-132.6
36	-46.23	-50.2
37	-47.99	-143.2
38	-49.65	-60.1
39	-51.05	-154.0
40	-52.66	-69.2
41	-54.02	-163.4
42	-55.50	-78.1
43	-56.82	-171.7
44	-58.21	-86.3
45	-59.50	-179.3
46	-60.82	-93.8
47	-62.06	173.6
48	-63.33	-100.6
49	-64.53	167.4